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## Algorithms of the phase-shifting interferometer via wavelength tuning

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**Abstract:** This paper describes the basic principle of wavelength scanning interferometer and analyzes its characteristics. Their merits and application fields are shown by comparing wavelength scanning interferometer with conventional hardware phase-shifting interferometer. For tunable semiconductor laser from American New Focus Company as example, hardware of realizing wavelength tuning is depicted simply. Algorithms of wavelength scanning interferometer are divided into three kinds: weighted multi-step phase shifting algorithm, the algorithm based on Fourier-transform technique and multi-wavelength algorithm. Their merits and application areas are pointed out through narrating and analyzing them in detail. Furthermore, a new algorithm based on Fourier-transform technique combining with different calculations is proposed, which is suitable to measure steps. The new algorithm overcomes the disadvantages of old one needing datum reference and plane of reference, increasing this algorithm's utility.

**Key words:** phase-shifting interferometry; wavelength tuning; phase-shifting algorithms

## 波长移相干涉仪的算法研究

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**摘要:** 叙述了波长移相干涉仪的基本原理, 分析了其特点, 对传统硬件移相干涉仪和波长移相干涉仪进行了比较, 指出了它们的优缺点和应用范围。以美国 New Focus 公司的可调谐半导体激光器为例, 简述了实现波长调谐的硬件。文中将波长移相干涉仪算法分成三类: 加权多步波长移相算法、基于傅里叶变换的波长移相算法和多波长算法, 对这三类算法进行了较详细的叙述和分析, 指出了各自的优缺点和应用范围。基于傅里叶变换的波长移相算法, 提出了结合差分运算的适合于台阶测量的新算法, 克服了已有算法中需要参考基准和参考面的缺点, 提高了算法的实用性。

**关键词:** 相移干涉术; 波长调谐; 移相算法

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### 1 Introduction

Phase-shifting interferometry is widely used in digital inspection due to its high precision and easy

practice. According to the realization methods of phase-shifting, it can be divided into two kinds: hardware phase-shifting and wavelength phase-shifting (also can be called quasi-software phase-shifting). In hardware phase-shifting, we realize

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the shifting through moving PZT, rotating half-wave plate, moving diffraction grating or tilting parallel flat, etc.<sup>[1]</sup>. Among them, the phase-shifting by moving PZT is applied widely.

Take phase-shifting by PZT for example. In practical using, it has the following problems: (1) When the measuring system has a large scale (for example, the diameter of reference mirror exceeds 500 mm), it is difficult to realize phase-shifting because of the high requirement for the PZT component. (2) Interferometers always use laser as source. For the laser's longer interferential length, parasitical fringes will be engaged in when measuring a good parallelism component. To eliminate the infection of these parasitical fringes, we must smear vaseline on the non-measuring plane, which is discommodious and will lead error to the result and which is bothersome especially in fast measurement or while measuring papery workpieces such as crystallitic vitreous plate.

Phase-shifting interferometry via wavelength tuning is a quasi-software shifting technology and the phase-shifting of the interferential signal is realized by altering the output wavelength. In the course of the phase-shifting, none components need moving. Its merits are as follows. (1) The fast respond decides its suit in larger interferential system. (2) It can predigest the structure of the interferometer. (3) It can eliminate the infection of parasitical fringe and realize the separated measurement of multi reflective surfaces. However, because the tunable diode laser is expensive and meanwhile the algorithms are immature, it is only suited in the situation where hardware phase shifting is unsuitable<sup>[2-10]</sup>.

The paper divides the main algorithms of phase-shifting interferometer via wavelength tuning into three kinds: the weighted multi-step phase-shifting algorithm<sup>[2,4,11]</sup>, the algorithm based on temporal Fourier transform<sup>[10]</sup> and the combined wavelength algorithm. The weighted multi-step phase-shifting algorithm is applied to test some precision optical wave-front. Meanwhile, this kind of

algorithm can restrain the infection of the parasitical fringes. The algorithm based on temporal Fourier transform is used to measure the profile with higher steps or slopes. And the combined wavelength algorithm is also applied to measure the course profile.

The overseas scientists have done some research and gotten good results. The ZYGO Co. U. S. A has produced the wavelength tuning interferometer with 600 mm diameter's measuring aperture. In China, Shanghai University and Chengdu Optical Precision Manufacturing Center is active in the study and exploitation. This paper will present the principle of the phase-shifting interferometry via wavelength tuning and summarize the main algorithms based on our experience.

## 2 Principle of phase-shifting interferometry via wavelength tuning

In Fig. 1, the OPD between the reference mirror and the measured surface is expressed as  $h(x, y)$  and the intensity of the interferential signal is as follows:

$$I_k(x, y) = A_0(x, y) \{ 1 + V_0(x, y) \cos[ \frac{4}{\lambda_k} h(x, y) ] \} = A_0(x, y) \left\{ 1 + V_0(x, y) \cos[ \frac{4}{\lambda_k} h(x, y) ] \right\}, \quad (1)$$

Where  $A_0(x, y)$  is the background,  $V_0(x, y)$  is the modulus and  $\lambda_k = \lambda_0 + k \Delta \lambda$ ,  $k = 0, 1, \dots, N - 1$ , is the wavelength of the  $k$ th phase-shifting. From the formula above, it can be seen that the phase-shifting is realized by wavelength tuning.

Compared with traditional hardware phase-shifting, phase-shifting via wavelength tuning has the following characteristics: (1) Phase-shifting value is correlative to  $h(x, y)$ . For different points whose  $h(x, y)$  are different, the phase-shifting values are also different for the same wavelength changing. (2) The linear wavelength changing will cause nonlinear phase-shifting value. These characters determine the difference between the wave-

length tuning algorithms and the hardware phase-shifting algorithms. The algorithms of phase-shifting interferometer via wavelength tuning will mainly solve nonlinear problem.

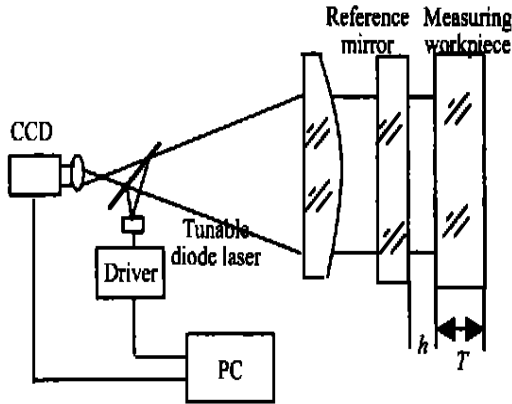


Fig. 1 Sketch map of phase-shifting interferometer via wavelength tuning

### 3 Hardware of wavelength tuning

The wavelength tuning is usually realized by the tunable diode laser. Fig. 2 is the wavelength tuning principle of velocity tunable diode laser (Mode 6 308), made by New Focus Co., USA.

A high-reflection coating on one end of the diode laser forms one end of the cavity and a high-reflecting tuning mirror forms the other. Starting from the diode the beam in the cavity passes through a collimating lens and then strikes a diffraction grating at near grazing incidence. The beam is diffracted toward the tuning mirror which reflects the light back on itself for the reverse path. Part of the light from the diode is reflected, not diffracted by the grating. This portion forms the output beam. The grating functions as a narrow spectral filter. Its passband is only a few GHz wide. The high wavelength selectivity results because many lines of the grating are illuminated by the grazing-incidence beam and because the beam is diffracted by the grating twice in each round trip through the cavity. The grating spectral filter is narrow enough to force the laser to operate in a single longitudinal mode. Different wavelengths diffract off the grating at different angles. However, only one wavelength leaves the grating in a di-

rection that is exactly perpendicular to the surface of the tuning mirror forming the resonant laser cavity. This is the lasing wavelength, because it's the only one that will survive for many cavity round trips. It follows then, that we can tune the laser by changing the angle of the tuning mirror.

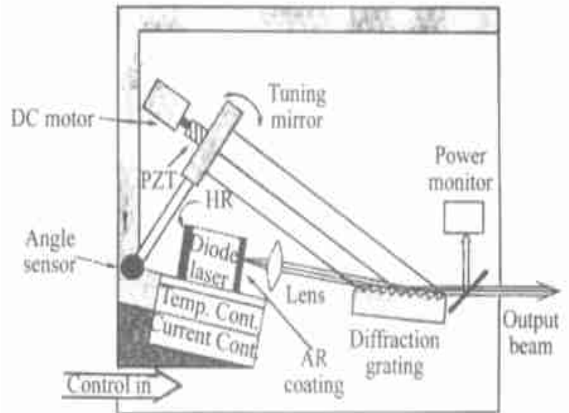


Fig. 2 Wavelength tuning principle of tunable diode laser

Through driving the DC Motor or PZT, the direction of the mirror can be changed to realize the wavelength tuning. The coarse adjustment, which is realized by driving the DC Motor, is 0.02 nm and the fine adjustment, which is realized by moving the PZT, is 0.001 nm (Higher resolution can be realized by externally connecting more exact voltage controller). In the 6 300 velocity tunable diode laser, it includes a controlling part to ensure the output steady power during wavelength changing, which is very important for precision measurement.

### 4 Main algorithms of phase-shifting interferometer via wavelength tuning

#### 4.1 Weighted multi-step phase-shifting algorithm via wavelength tuning<sup>[2,4,11]</sup>

Weighted multi-step phase-shifting algorithm via wavelength tuning succeeds the idea of hardware phase-shifting algorithm and the core is using algorithm to minish the nonlinear error produced by the phase-shifting principle itself. In phase-

shifting interferometer via wavelength tuning , the changing frequency of the interferential signal is correlative to the OPD between the two interferential beams. Because the different OPD between the interferential signal and parasitical signal leads to different frequency changing corresponding to the same wavelength shifting , we can use signal processing theory to separate them. By phase-shifting interferometry via wavelength tuning , we can realize not only the detection of the profile but also the measurement of the thickness variation or the refractive index of optics.

In the wavelength tuning interferometer , we assume the phase of the initiative interferential signal is:

$$0(x, y) = \frac{4}{0} \frac{h(x, y)}{0} , \quad (2)$$

Then the phase of the  $k$  th interferential signal is

$$k(x, y) = \frac{4}{0+k} \frac{h(x, y)}{0} - \frac{4}{0} \frac{h(x, y)}{2} k = 0(x, y) - 2 \nu(x, y) k \quad (k = 0, 1, \dots, N - 1) , \quad (3)$$

So the temporal frequency of the interferential signal is

$$\nu(x, y) = \frac{2h(x, y)}{2}$$

In Fig. 3 , each frequency in the measurement can be described as follows<sup>[4]</sup>

$$\nu_{p, q} = \frac{2}{0} [ ph + qn_0 T(1 - \frac{0}{n_0} \frac{dn}{d}) ] , \quad (4)$$

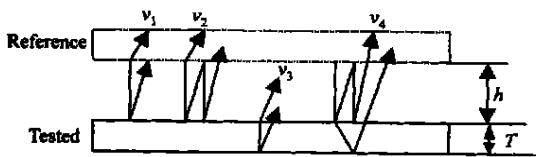


Fig. 3 Sketch map of the measuring information

Where  $p, q$  are both integers reflecting the paths of interferential beams ,  $n_0$  is the refractive index of the measured components when  $= 0$  and  $T$  is the thickness of the workpiece.

It is assumed that  $\frac{n_0 T}{h} = 3$  , measuring material is BK7 , initiative wavelength is 690 nm and  $\frac{0}{n_0} \frac{dn}{d} = 1.2\%$  (For BK7). Table 1 shows the frequency information of the interferential signals. Except profile and thickness information , other information is looked as parasitical signals in the measurement. We can separate the useful signal from other parasitical signals applying the signal analysis theory.

Tab. 1 Frequency information of each measuring signal

No.	$\nu_k / \nu_1$ ( $\frac{0}{n_0} \frac{dn}{d} = 0$ )	$\nu_k / \nu_1$ (For BK7)	Remark
$\nu_1$	1 ( $p = 1, q = 0$ )	1	profile
$\nu_2$	2 (2, 0)	2	
$\nu_3$	3 (0, 1)	3.04	thickness
$\nu_4$	2 (-1, 1)	2.04	

According to the signal analysis theory , we design a 19-step algorithm. The phase of point  $(x, y)$  is:

$$0(x, y) = \arctan \frac{\sum_{k=1}^{19} s_k I_k(x, y)}{\sum_{k=1}^{19} c_k I_k(x, y)} , \quad (5)$$

While measuring the profile ,  $s_k = w(k) \sin(\frac{0}{6} \times k)$  ,  $c_k = w(r) \cos(\frac{0}{6} \times k)$ .

While measuring the thickness ,  $s_k = w(r) \sin(\frac{0}{2} \times k)$  ,  $c_k = w(r) \cos(\frac{0}{2} \times k)$ .

In the formula above ,  $w(k)$  is the weighted values determined by the window function.

There are many kinds of window function such as Bartlett , Blackman , Hanning and Lifting Cosine etc. In order to test the characteristic of the window function ,people define the following functions to estimate the restraint of harmonic<sup>[2]</sup>

$$F_1(\nu) = \sum_{k=1}^{19} s_k \exp(-i k \frac{\nu}{\nu_1}) , \quad (6)$$

$$F_2(\nu) = \sum_{k=1}^{19} c_k \exp(-i k \frac{\nu}{\nu_1}) , \quad (7)$$

Take Hanning window for an example, its figure and evaluating function are showed in Fig. 4 ~ 6. In three figures, it is assumed that  $n_0 T/h = 3$  and we do not consider the infection of reflection coefficient

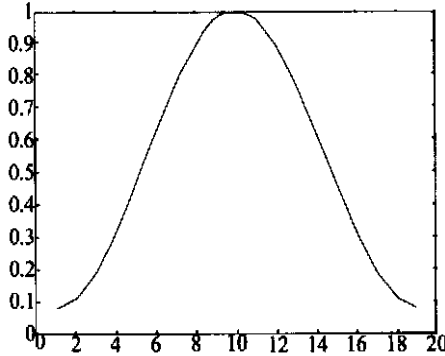


Fig. 4 Hanning Window

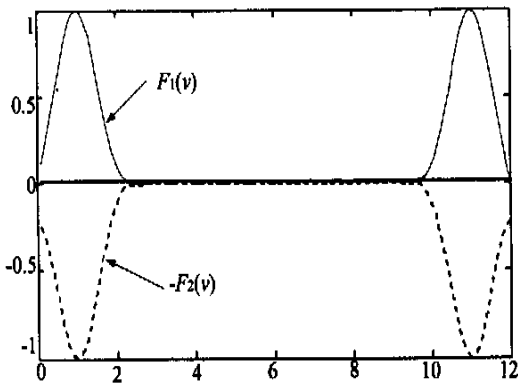


Fig. 5 Evaluating function of 19-step algorithm in profile measurement

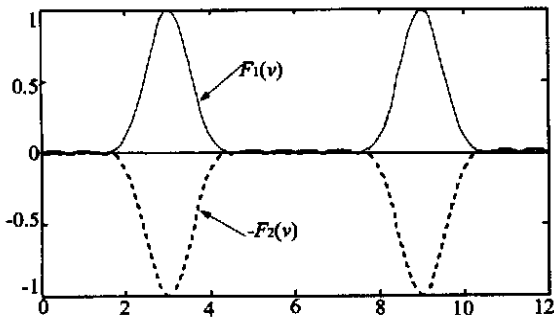


Fig. 6 Evaluating function of 19-step algorithm in thickness measurement

It can be seen that the useful and the useless signals can be separated available. It is one of the

shortcomings of the phase-shifting algorithm via wavelength tuning that the thickness of the workpiece and  $h$  must fit certain terms, i. e. the workpiece must be suited in a certain position. Normally,  $2 \frac{n_0 T}{h} = 5$ .

From the figures above, it also can be seen that the sensitivities of the algorithm for some harmonious signals are unequal to zeros, which will cause some rudimental errors and effect the testing result. Therefore, the window function usually needs adjusting ulteriorly. In order to improve the restrain characteristic, the weighted values determined by window function are usually processed ulteriorly, but the processed results should satisfy<sup>[11]</sup>

$$\begin{aligned} \sum_{k=1}^N s_k = 0 \quad \sum_{k=1}^N c_k = 0 \\ \sum_{k=1}^N s_k \sin(-k) = \sum_{k=1}^N c_k \cos(k) \\ \sum_{k=1}^N s_k \cos(k) = \sum_{k=1}^N c_k \sin(-k), \end{aligned} \quad (8)$$

Where  $k$  is the phase-shifting value.

According to the principle described above, we can define more algorithms suitable for the actual interferometer.

### 4.2 Phase-shifting algorithm based on temporal Fourier transform

Tuning wavelength for several times, we acquire  $N$  interferograms shown in Fig. 7. Each points has  $N$  samples in temporal space. For a series of intensities of each point we do Fourier transform and take out a frequency chart through a filter window which is shown in Fig. 8, then we do inverse Fourier transform and phase unwrapping, we can acquire the distribution of phase  $k(x, y)$ . Because of the periodicity of the cosine signal and the processing of phase unwrapping,  $k(x, y)$

$k(x, y)$  (see equation (1)) :

$$k(x, y) + \text{constant} = k(x, y) = \frac{4}{k} h(x, y), \quad (9)$$

The constant is unknown, so from  $k(x, y)$ ,

$h(x, y)$  can not acquired directly.

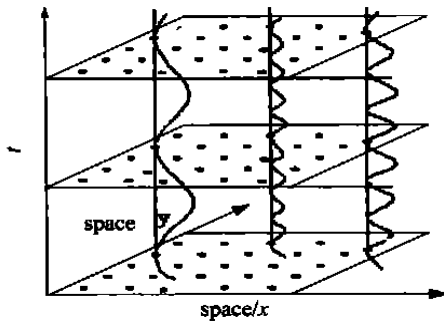


Fig. 7 Temporal sampling

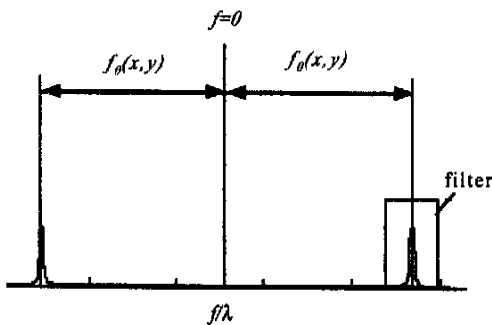


Fig. 8 Fourier transform of signals

To resolve this problem, Takeda and his colleagues proposed applying a reference piece with a known height and a datum plate in the testing system as the contrastive object<sup>[10]</sup>, shown in Fig. 9.

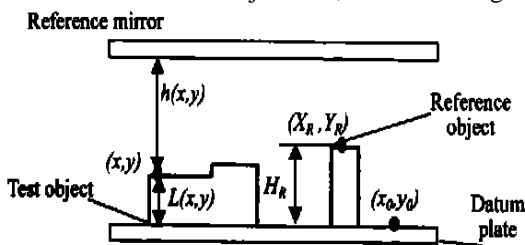


Fig. 9 Principle of Takeda algorithm

The measured profile is

$$L(x, y) = - H_R \times \left\{ \frac{\partial}{\partial k} [ I_k(x, y) - I_k(x_0, y_0) ] \right\}, \tag{9}$$

where  $\{ \}$  presents the average calculation.

This algorithm is suitable for testing some course profiles or the work piece with high steps.

The shortcoming of this algorithm is obvious. It needs a reference object with a proper known height and a datum plate with a good flatness. In order to improve this algorithm and to make it more useful, the paper puts forward a new algorithm.

Although,  $I_k(x, y)$  and  $I_{k-1}(x, y)$  their differences are equal, namely  $\nabla_k I_k(x, y) = \nabla_{k-1} I_{k-1}(x, y)$ , so

$$h(x, y) = \frac{\sum_{k=1}^{N-1} \left\{ \nabla_k I_k(x, y) \left[ \frac{4}{0+k} - \frac{4}{0+(k-1)} \right] \right\}}{\sum_{k=1}^{N-1} \left[ \frac{4}{0+k} - \frac{4}{0+(k-1)} \right]^2}, \tag{10}$$

Obviously, the proposed algorithm avoids the shortcomings of the Takeda's algorithm. It is suitable to measure the profile with steps similarly.

### 4.3 Combined wavelength algorithm

In phase shifting interferometry, because of the periodicity of interferential signals, the changing phase of the adjacent points should less than  $2\pi$ , i.e. the changing profile of the adjacent points is less than  $\lambda/2$ , which limits the measuring range of multi-step phase-shifting algorithm. There are two methods to enlarge the range. One is using the light source with longer wavelength but also has a bound. The other is using multi-wavelength combination method. For two-wavelength combined algorithm as example.

When laser wavelength is  $\lambda_1$ , the phase of interferential signal is  $\phi_1(x, y) = \frac{4\pi}{\lambda_1} h(x, y)$ . Then shift the wavelength to  $\lambda_2$ , we get  $\phi_2(x, y) = \frac{4\pi}{\lambda_2} h(x, y)$ . Their difference is

$$\Delta\phi(x, y) = \phi_1(x, y) - \phi_2(x, y) = \frac{4\pi}{\lambda_2 - \lambda_1} h(x, y), \tag{11}$$

For  $\frac{\lambda_2 - \lambda_1}{\lambda_1} \gg 1$ ,  $\lambda_2$ , the depth measuring range of multi-step phase-shifting algorithm makes great increase. For example,  $\lambda_1 = 670 \text{ nm}$  and  $\lambda_2 = 671 \text{ nm}$ , then  $\frac{\lambda_2 - \lambda_1}{\lambda_1} = 449.570 \text{ nm}$ . The jump changing of adjacent points adopting single-wavelength measurement and double-wavelength com-

bined measurement are relatively less than  $\lambda/2 = 335 \text{ nm}$  and  $\frac{\lambda_1 - \lambda_2}{2} = 224.785 \text{ nm}$ . The smaller wavelength interval, the larger realizable measuring range. Since the easy realization of the wavelength shifting which makes combined wavelength measurement easily realize, using combined wavelength measurement can enlarge the measurement range of phase-shifting algorithm. This algorithm is suitable for the measurement of higher step.

## 5 Conclusion

The characters of phase-shifting interferometry via wavelength tuning are (1) the phase shifting value is correlative to the OPD, i. e. phase shifting value of each spatial points is different even though the wavelength changing is the same. (2) The nonlinear error of phase shifting via wavelength tuning leads the main error of measurement. (3) It is suitable to the interferometer with large scale for its

fast respond and low demand for the interferometer's structure. (4) It can realize the separation of multi-signals. The phase-shifting algorithms via wavelength tuning involve three kinds: weighted multi-step phase shifting algorithm, phase-shifting algorithm based on temporal Fourier transform and combined wavelength algorithm. Weighted multi-step phase shifting algorithm can restrain the harmonic error led by the nonlinear phase shifting and be able to realize the separation of multi-signals. The shortcoming of this algorithm is that it has limit for the position of the tested workpiece. This algorithm is adapted to measure minute profile. Phase-shifting algorithm based on temporal Fourier transform can calculate the absolute value of the OPD. It is suitable for measuring rough or stepped profiles. Combined wavelength algorithm is easy to realize and is also fit to measure rough or stepped profiles. In a word, the phase-shifting interferometry via wavelength tuning is in progress.

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